

# Math Olympiad Mathlete National Selection Test For Secondary School 2021

**Junior High**  
Time: 90 minute

Jointly Organized by:  
Persatuan Matematik Olympiad Malaysia (PERMATO)  
E Mathematics Olympiad System (EMOS)

## Information for candidates:

1. Do not open the booklet until told to do so by the invigilator.
2. Write the answers in the space provided.
3. Answer all questions, each question carries 4 mark. For problems involving more than one answer, full credit will be given only if ALL answers are correct, no partial credit will be given. There is no penalty for a wrong answer.
4. Answer the problems with pencil, blue or black pen.
5. The diagrams in the questions provided are not drawn to scale.
6. The use of calculator is prohibited.
7. All papers shall be collected at the end of this test.

Name:	Grade:
School:	Score:

Question	Answer
1. 有几个二位数数字可被其数字的乘积整除? How many two digit numbers divisible by the product of the digits?	5
2. 已知 $(a - 5)^2 + (b - c)^2 + (c - d)^2 + (b + c + d - 9)^2 = 0$ , 问 $(a + b + c)(b + c + d) = ?$ Given $(a - 5)^2 + (b - c)^2 + (c - d)^2 + (b + c + d - 9)^2 = 0$ , then $(a + b + c)(b + c + d) = ?$	99
3. $x^4 - y^4 = 15, \forall x, y \in \mathbb{Z}^+$ . 求 $x^4 + y^4$ . $x^4 - y^4 = 15, \forall x, y \in \mathbb{Z}^+$ . Find the value of $x^4 + y^4$ .	17
4. 若 $\sqrt{9 - (n + 2)^2}$ 为一实数, 其中 $n$ 为整数。试问 $n$ 有几个可能值? For how many integers $n$ is $\sqrt{9 - (n + 2)^2}$ a real number.	7
5. 一凸多边形内角和为 $2190^\circ$ (其各内角小於 $180^\circ$ )。问此多边形共有几条边? The sum of all angle except one of a convex polygon is $2190^\circ$ , (where the angles are less then $180^\circ$ ). Find the number of sides of the polygon.	15
6. 计算 $100 - 1 + 99 - 2 + 98 - 3 + \dots + 52 - 49 + 51 - 50$ Evaluate $100 - 1 + 99 - 2 + 98 - 3 + \dots + 52 - 49 + 51 - 50$	2500
7. Rt $\triangle ABC$ , $\angle A = 90^\circ$ 其中 $O$ 为其内心。 $O$ 至 $BC$ 的垂距为 $\sqrt{8}$ 。求 $AO$ 的长度。 Rt $\triangle ABC$ , $\angle A = 90^\circ$ and $O$ is the incentre. The perpendicular distance of $O$ from $BC$ is $\sqrt{8}$ . Find the length of $AO$ .	4
8. 方程式 $x^2 - 2ax + a^2 + a - 3 = 0$ 有实数根且其值小於 3。下列何者为真? If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3, then A $a < 2$ B $2 \leq a \leq 3$ C $3 \leq a \leq 4$ D $a > 4$	A
9. $\triangle ABC$ , 点 $E$ 在 $AB$ 上, 且 $AC = AE$ 和 $EC = EB$ 。已知 $\angle CAB = 80^\circ$ 。求 $\angle ECB$ 。 $\triangle ABC$ , $E$ is the point on side $AB$ , $AC = AE$ and $EC = EB$ . If $\angle CAB = 80^\circ$ . Find $\angle ECB$ .	25
10. 将 8 颗不同的糖果分给 3 位小朋友且每位小朋友至少得 1 颗糖果共有几种分法? How many different ways can 3 children share 8 identical sweets so that each child get at least one?	21
11. 求 $(2006)^{2005}$ 的最小两位数。 Find the last two digits of $(2006)^{2005}$ .	76

Question	Answer										
<p>12. 已知 <math>\frac{97}{19} = a + \frac{1}{b + \frac{1}{c}}</math>, <math>a, b, c \in \mathbb{Z}</math>. 求 <math>a + b + c</math>.</p> <p>If <math>\frac{97}{19} = a + \frac{1}{b + \frac{1}{c}}</math>, <math>a, b, c \in \mathbb{Z}</math>. Find <math>a + b + c</math>.</p>	16										
<p>13. <math>x, y</math> 为正数且 <math>xy = 1</math>. 求 <math>\frac{1}{x^4} + \frac{1}{y^4}</math> 的最小值。</p> <p>The positive number <math>x</math> and <math>y</math> satisfy <math>xy = 1</math>. Find the minimum value of <math>\frac{1}{x^4} + \frac{1}{y^4}</math>.</p>	2										
<p>14. <math>\frac{36^{24}}{24^{36}}</math> 可化简为 <math>(\frac{3}{a})^b</math>, 求 <math>a + b</math> 之值。</p> <p>Given <math>\frac{36^{24}}{24^{36}}</math> can be simplify as <math>(\frac{3}{a})^b</math>. Find <math>a + b</math>.</p>	44										
<p>15. 已知 <math>a_1 = a_2 = 1, a_i = \min\{a_{i-1}, a_{i-2}\}</math> 当 <math>i &gt; 2</math>. 求 <math>a_{2006}</math>.</p> <p>For <math>a_1 = a_2 = 1, a_i = \min\{a_{i-1}, a_{i-2}\}</math> for <math>i &gt; 2</math>. Find <math>a_{2006}</math>.</p>	1										
<p>16. 求 <math>x + \sqrt{x^2\sqrt{x^3 + 1}} = 1</math> 的实数解。</p> <p>Find the number of real solution of the equation <math>x + \sqrt{x^2\sqrt{x^3 + 1}} = 1</math></p>	1										
<p>17. 求 <math>(999999999)^3</math> 各个位数数字之和。</p> <p>Find the sum of the digits of <math>(999999999)^3</math></p>	180										
<p>18. 一正五边形的内角与一正十边形的外角比例为 <math>x:1</math>. 求 <math>x</math>.</p> <p>The ratio of an interior angle of a regular pentagon to an exterior angle of a regular decagon <math>x:1</math>. Find <math>x</math>.</p>	3										
<p>19. <math>\alpha</math> 为一质数, 使得 <math>\alpha^2 - 8\alpha - 65 &gt; 0</math>. 求 <math>\alpha</math> 的最小值。</p> <p><math>\alpha</math> is a prime number such that <math>\alpha^2 - 8\alpha - 65 &gt; 0</math>. Find the smallest value of <math>\alpha</math>.</p>	17										
<p>20. 乌托邦的货币 5Wu 等于 3Bi, 10Bi 等于 2Ha. 问 25Wu 等于多少 Ha?</p> <p>In the currency of Utopia, 5Wu are equal to 3Bi, and 10Bi are equal to 2Ha. How many Ha equal to 25Wu?</p>	3										
<p>21. 一三角形的三边边长比例为 13:14:15, 其周长为 84cm. 求此三角形的面积 (<math>\text{cm}^2</math>).</p> <p>The sides of a triangle are in ratio of 13:14:15 and its perimeter is 84cm. Determine the area of the triangle in <math>\text{cm}^2</math>.</p>	336										
<p>22. 共有 400 名学生参加一数学比赛。比赛试卷满分为 100 分。他们的得分统计表如下表。</p> <p>问任意从中抽出一学生的试卷且得分高于 50 分的几率为多少? (以小数表示)</p> <p>400 students of a math competition appeared for a test of 100 mark. The data about the marks secured is presented in the table below. Randomly pick up a result card, what is the probability that the student has secured more than 50 marks? (in decimal representation)</p> <table border="1" style="width: 100%; margin-top: 10px;"> <tr> <td>Marks secured 得分</td> <td>2 ~ 25</td> <td>26 ~ 50</td> <td>51 ~ 75</td> <td>&gt;75</td> </tr> <tr> <td>Number of students 学生数量</td> <td>50</td> <td>220</td> <td>100</td> <td>30</td> </tr> </table>	Marks secured 得分	2 ~ 25	26 ~ 50	51 ~ 75	>75	Number of students 学生数量	50	220	100	30	0.325
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Question	Answer								
<p>23. <math>\alpha, \beta, \gamma, \delta</math> 为 4 相异实数且 <math>\alpha &gt; \beta</math>, <math>\gamma &lt; \delta</math>, <math>\beta &gt; \gamma</math>, <math>\delta &lt; \alpha</math>。下列何者为真?  <math>\alpha, \beta, \gamma, \delta</math> are four distinct positive real numbers such that <math>\alpha &gt; \beta</math>, <math>\gamma &lt; \delta</math>, <math>\beta &gt; \gamma</math>, and <math>\delta &lt; \alpha</math>. Then</p> <table border="1" data-bbox="196 472 533 723"> <tr> <td>A</td> <td><math>\beta &lt; \delta &lt; \alpha</math></td> </tr> <tr> <td>B</td> <td><math>\delta &gt; \gamma</math> 且 <math>\delta &lt; \beta</math> <math>\delta &gt; \gamma</math> and <math>\delta &lt; \beta</math></td> </tr> <tr> <td>C</td> <td><math>\delta &lt; \beta &lt; \alpha</math></td> </tr> <tr> <td>D</td> <td><math>\alpha</math> 为 4 数中最大数 <math>\alpha</math> is the greatest.</td> </tr> </table>	A	$\beta < \delta < \alpha$	B	$\delta > \gamma$ 且 $\delta < \beta$ $\delta > \gamma$ and $\delta < \beta$	C	$\delta < \beta < \alpha$	D	$\alpha$ 为 4 数中最大数 $\alpha$ is the greatest.	<p><b>D</b></p>
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C	$\delta < \beta < \alpha$								
D	$\alpha$ 为 4 数中最大数 $\alpha$ is the greatest.								
<p>24. 有一二位数数字之值增加 20%后, 其个位数数字与十位数数字恰好颠倒。求此二位数数字的各位数数字之和。  A two digit number is increased by 20%, when its digits are reversed. Find the sum of the digits of the number.</p>	<p><b>9</b></p>								
<p>25. 已知 <math>x + y + z = 2007</math>, <math>xy + yz + zx = 4011</math>, 且 <math>x, y, z \neq 1</math>。  求 <math>\frac{1}{1-x} + \frac{1}{1-y} + \frac{1}{1-z}</math>。  For <math>x + y + z = 2007</math>, <math>xy + yz + zx = 4011</math>, and <math>x, y, z \neq 1</math> Find the value of <math>\frac{1}{1-x} + \frac{1}{1-y} + \frac{1}{1-z}</math>.</p>	<p><b>0</b></p>								