PERMATO
Persatuan Matematik Olympiad Malaysia

数学奥林匹克学研中心
E MATHEMATICS OLYMPIAD SYSTEM

Math Olympiad Mathlete National Selection Test For Secondary School 2021

Senior High

Time: 90 minute

Jointly Organized by:
Persatuan Matematik Olympiad Malaysia (PERMATO)
E Mathematics Olympiad System (EMOS)

Information for candidates:

1. Do not open the booklet until told to do so by the invigilator.
2. Write the answers in the space provided.
3. Answer all questions, each question carries 4 mark. For problems involving more than one answer, full credit will be given only if ALL answers are correct, no partial credit will be given. There is no penalty for a wrong answer.
4. Answer the problems with pencil, blue or black pen.
5. The diagrams in the questions provided are not drawn to scale.
6. The use of calculator is prohibited.
7. All papers shall be collected at the end of this test.

Name:	Grade:
School:	Score:

This Question Paper Consists of 5 Printed Pages

	Question	Answer
1.	Fatimah 需要设法背下她的 2 位数字分机号码。她知道该数可被 5 整除且个位数是一奇数。请问可能为她分机号码的数字有几个? Fatimah tries to remember her 2-digit extension number. She knows that the number is divisible by 5 and that the first digit is odd. How many possibilities are there for this number?	10
2.	Muhammad 选用的通讯商采统一收费方式。若 Muhammad 通话 11 分钟被通讯公司征收 RM4.88，通话 19 分钟则被征收 RM6.00。问 Muhammad 通话 15 分钟将被征收多少令吉? Muhammad phone company charge him a per-minute charge a well as a connection fee (which is the same for every call) every time he makes a phone call. If Muhammad charged RM4.88 for an 11-minute call and RM6.00 for a 19-mintute call, how much would he be charged for a 15-minute call?	5.44
3.	假设$n \in \mathbb{Z}^+$, s_n 为首 n 个最小质数之和. 问当s_n为一完全平方数时n的最小可能值为何? Let $n \in \mathbb{Z}^+$, s_n be the sum of the n smallest primes. Find the least n such that s_n is a perfect square.	9
4.	求 2011^{2011} 除 7 的余数。 Find the remainder when 2011^{2011} is divided by 7.	2
5.	Boleh 篮球联盟共有 30 队篮球队。若要从中选出选出 4 队出来举行比赛且 Tiger Internationals 队与 Hebat United 队不可同时被选中，共有几种选法? There are 30 basketball teams in Boleh Basketball League. In how ways can 4 teams be chosen for a tournament if the two teams Tiger Internationals and Hebat United cannot be chosen at the same time?	27027
6.	将 1，2，3，4，5，6 任意写在一圆的圆周上。求其中任意 4 数字中的中间两数之和小於另两数之和的机率。 The numbers 1, 2, 3, 4, 5, 6 are randomly written around a circle. What is the probability that there are four neighboring numbers such that the sum of the middle two numbers is less then the sum of other two?	1
7.	求$\frac{4x^2+8x+13}{6(1+x)}$ 且 $x \geq 0$ 的最小值 Find the smallest value of $\frac{4x^2+8x+13}{6(1+x)}$ for $x \geq 0$	2
8.	$\Delta ABC, \angle A = 130^o, BC = 6$cm. 求其外接圆半径（cm）。 $\Delta ABC, \angle A = 130^o, BC = 6$cm. Find the radius of the circum circle (in cm).	6
9.	一正方形与一三角形的边皆无平行关系，问他们最多能有几个交点? Find the maximum number of points of intersection between a square and a triangle. (no side of the triangle is parallel to any side of square).	6
10.	一等腰三角形的两个角分别为 80^o and x^o。求x的所有可能值之和。 Two angles of an isosceles triangle measure 80^o and x^o. What is the sum of all the possible values of x?	150

	Question	Answer
11.	a， b，c 和 d 为一正多边形的连续 4 个边。已知 $\angle ACD = 120^{o}$, 求此正多边形的边数。 Let a, b, c and d be consecutive vertices of a regular polygon. If $\angle ACD = 120^{o}$, how many side does the polygon have?	9
12.	一元方程式为$x^2 + y^2 = 100$，点(a, b)为该圆上的点且 $a, b \in \mathbb{Z}$。问共有几个点(a, b)满足以上条件? Given the equation of the circle $x^2 + y^2 = 100$, the number of points (a, b) lying on the circle, where $a, b \in \mathbb{Z}$ is ____.	12
13.	有n个不重叠的圆在同一平面上，他们的的半径分为 1， 2， … ， n，且$n \in \mathbb{Z}^+$。已知它们的面积和至少为 100，求n的最小值。 There are n non-overlapping circles in a plane with radii 1, 2, … , n, where $n \in \mathbb{Z}^+$. The total area that they enclose is at least 100. Find the minimum possible value of n.	5
14.	锐角三角形ABC, D点 和 E点 在 ΔABC 内，使得$DE \| \| BC$。比起E点，B点较接近D点，已知 $\angle AED = 80^{o}$, $\angle CBD = 40^{o}$。 求 $\angle BAE$。 In acute triangle ABC, D and E are points inside triangle ABC such that $DE \| \| BC$, B is closer to D than it is to E, $\angle AED = 80^{o}$, $\angle CBD = 40^{o}$. Find the measure of $\angle BAE$, in degrees.	50
15.	若一数的数字按相反的顺序重新排列后，所得到的数和原来的数一样，此数称之为回文数。例：1，11 和 141 都是回文数。问 1 至 1000 间有多少个可被 11 整除的回文数? A palindrome is a number that reads the same forwards and backwards. For example, 1, 11 and 141 are all palindromes. How many palindromes between 1 and 1000 are divisible by 11?	17
16.	设x, y, z 为实数且满足下列方程组: $$x + y - z = 5$$ $$y + z - x = 7$$ $$z + z - y = 9$$ 求 $x^2 + y^2 + z^2$。 Suppose x, y, z are real numbers that satisfy: $$x + y - z = 5$$ $$y + z - x = 7$$ $$z + z - y = 9$$ Find $x^2 + y^2 + z^2$.	149
17.	Ah Ming 有一六面骰子，此骰子每一面的数与其对面的数之和均相等。若 109，131，135 分别写在一共用同一个顶点的三个面上。若另外三面所写的数字均为小于 200 的质数，求这三数之和的最大值。 Ah Ming has a six-sided dice with a number written on each face that the sums of the numbers written on each pair of opposite faces are equal to each other. Suppose that the numbers 109, 131, and 135 are written on three faces which share a corner. Determine the maximum possible sum of the numbers on this three remaining faces, give that all three positive primes less than 200.	39

	Question	Answer
18.	Abu 计划在进入森林冒险前购买便当。Abu 准备用他拥有的两张不同的优惠卷来购买一个原价为 RM20 的便当。其中一张优惠卷的优惠为折扣 RM3，另一张则是折扣当前价格的 10%。两张优惠卷可同时使用，但使用顺序需自行排序。问阿布最少使用多少钱购得一个便当？ Abu buying a lunch box before the ventures into the forest. The original cost of a lunch box is RM20, but Abu has two coupons. One coupon is RM3 off and the other is 10% off the entire remaining cost. Abu can use the coupons in any order. What is the least amount of the money he could pay for the lunch box?	15
19.	将 6 个不同的泰迪熊分给 Steven 和 Yuni，在 Steven 获得一蓝色泰迪熊与 Yuni 获得一绿色泰迪熊的情况底下，共有几种分配方法？（他们不需要获得相同数量的泰迪熊，但每个泰迪熊都必须分配出去） Steven and Yuni have 6 distinct teddy bears to split between them, including exactly 1 blue bear and 1 green teddy bear. How many ways are there for the two divide the teddy bears, if Steven gets the blue teddy bear and Yuni gets the green teddy bear? (The two do not necessarily have to get the same number of teddy bears, but each teddy bear must go to a person.)	16
20.	一青蛙从原点出发，共跳跃了 4 次，但最终还是回到原点。每一次跳跃只能向左或向右移动一个单位。问此青蛙有多少种跳跃的方式？ A frog is located at the origin. It makes 4 hops, each of which moves it either 1 unit to the right or 1 unit to the left. If it also ends at the origin, how many 4-hop path can it take?	6
21.	Sammy 所加入的数学学会有 6 名会员（含她自己）。Sammy 每天都与会员们一起外出用餐，每次每位与 Sammy 外出的会员都会给 Sammy RM1。问当 Sammy 与所有不同组合的会员外出后一共能得到多少令吉？ Including Sammy, there are six students in the math club. Each day, Sammy hangs out with a different group of club mates, each of whom give him RM1 when she hangs out with them. How many ringgit will Sammy have by the time she has hung out with every possible group of club mates?	80
22.	There are seven boxes in a line: three empty, three holding RM10 each, and one holding the jackpot of RM100000. From the left to right, the boxes are numbered 1, 2, 3, 4, 5, 6, and 7, in that order. You are told the following: • No two adjacent boxes hold the same contents. • Box 4 is empty. • There is one more RM10 prive to the right of thr jackpot than there is to the left Which box holds the jackpot? A. Box 1 B. Box 3 C. Box 6 D. Box 7 E. None of above	B

	Question	Answer
23.	$\Delta ABC, \angle ABC = 45^o$ and $\angle ACB = 60^o$。P点与Q点位于线段BC，F点位于线段AB，E点位于线段AC，且$FQ \| \| AB$。从顶点A做一垂线至BC交于D点。线段AD，FQ和PE将形成一三角形，找出此三角形所有可能的最大角与最小角之差的和。 In $\Delta ABC, \angle ABC = 45^o$ and $\angle ACB = 60^o$. Let P and Q be the points on segment BC, F a point on segment AB, and E a point on segment AC such that $FQ \| \| AB$. Let D be the foot of the altitude from A to BC. The lines AD, FQ and PE form a triangle. Find the positive difference, in degrees, between the largest and smallest angles of this triangle.	75
24.	已知$f(n) = \left\lfloor \frac{n}{2} \right\rfloor + f(\left\lfloor \frac{n}{2} \right\rfloor)$， n 为大于 1 的正整数。若$f(1) = 1$, k 为一小于或等于 2011 的正整数，求$f(k) = k$的最大值。 Given $f(n) = \left\lfloor \frac{n}{2} \right\rfloor + f(\left\lfloor \frac{n}{2} \right\rfloor)$ for every integer n greater than 1. If $f(1) = 1$, find the maximum value of $f(k) = k$, where k is a positive integer less than or equal to 2011.	529
25.	在一$n \times n$的方格表上有若干个n^2大小的正方格（$n \in \mathbb{Z}^+$），其中每一个正方格被涂成红色或蓝色且每行与每列只有 10 个蓝色正方格。若我们每一次在此方格表内任意选择一行（或列）并改变该行（或列）中所有方格的颜色（若方格为红色，我们将其改为蓝色；若方格为蓝色，我们将其改为红色）。重复此动作若干次后我们将会使得表格里拥有少于 10 n个蓝色正方格，求n的最大值。 An $n \times n$ square grid is formed by n^2 unit square where $n \in \mathbb{Z}^+$. Each unit square is then colored either red or blue such that each row or column has exactly 10 blue squares. A more consists of choosing a row or column, and recolor each each unit square in the chosen row or column: If it is red, we recolor it blue, and if it is blue, we recolor it red. Suppose that it is possible to obtain fewer than $10n$ blue squares after a sequence of finite number of moves. Find the maximum possible value of n.	39